Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713926090

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To cite this Article Kini, U. D.(1990) 'Discontinuous orientational changes in nematics Effects of electric and magnetic fields', Liquid Crystals, 8: 6, 745 — 763
To link to this Article: DOI: 10.1080/02678299008047386
URL: http://dx.doi.org/10.1080/02678299008047386

LIQUID CRYSTALS, 1990, VOL. 8, NO. 6, 745–763

Discontinuous orientational changes in nematics
Effects of electric and magnetic fields

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(*Received 21 March 1990; accepted 11 July 1990*)

Theoretical studies are presented on discontinuous changes produced in homogeneous deformations under the competing influences of an electric field **E** applied parallel to the sample planes and a magnetic field **H** applied at different angles in the plane containing **E** and $\mathbf{n}_0$ the initial director orientation. Orientational bistability can be affected by the tilt of $\mathbf{n}_0$ at the boundaries, the relative magnitudes of **H** and **E**, the tilt angle of **H** and even the order in which **E** and **H** are applied to the sample. In particular, the bend transition can be made continuous by applying **H** parallel to **E** in materials with positive diamagnetic susceptibility anisotropy ($\chi_a > 0$). Depending upon the tilt of **H** and the relative strengths of **H** and **E** it may be possible to change (continuously or otherwise) the nature of distortion in domains. *Ab initio* calculations show that the **E** induced twist transition should be continuous in samples having lateral dimensions large compared with thickness. In such a sample one can envisage the possibility of removing the discontinuity at the Freedericksz transition by tilting $\mathbf{n}_0$ sufficiently away from the homeotropic in a plane normal to **E**. A more complete theoretical analysis may be necessary to understand the discontinuous twist Freedericksz transition observed experimentally when the electrode gap is not large compared to the sample thickness. The possibility that the hysteresis width of a discontinuous Freedericksz transition might depend on domain formation above threshold cannot be ruled out. In the bend geometry non-rigid anchoring of the director at the boundaries is briefly dealt with to indicate that flexoelectricity might play a role in the initial build up of low level deformation prior to the transition when **E** is static. The flexoelectric contribution may become negligible in the presence of a sufficiently strong stabilizing **H** when the anchoring is strong enough and **E** is time varying. On the basis of the present results and previous work it may be concluded that variation of tilt of **H** may have considerable influence on the formation of the modulated phase above the bend transition.

1. Introduction

The effects of **E** and **H** on the orientation of nematics have been well understood on the basis of the continuum theory [1–7]. In particular, the study of the Freedericksz transition has enabled estimation of the curvature elastic constants K_1, K_2, K_3 of nematics and also made possible the development of a variety of field effect displays.

The interaction of **H** with a nematic occurs via the relatively weak χ_a. The interaction of **E** occurs via the dielectric anisotropy $\varepsilon_a (= \varepsilon_\parallel - \varepsilon_\perp$ where $\varepsilon_{\parallel,\perp}$ are the dielectric constants along and normal to the director, respectively) so that the induced polarization can considerably alter the field inside the sample [5] especially in the presence of a spatially varying director field. In addition, flexoelectricity [8] which is recognized as a material property of all varieties of nematics [9] may also assume importance [10]. Still, like the **H** induced Freedericksz transitions, the **E** induced transitions have generally been known to be of second order.

Recently, Frisken and Palffy-Muhoray [11, 12] have reported experimental observation of discontinuous **E** induced bend and twist transitions in a nematic (5CB) having high, positive ε_a. Using the analytical formulation of [14] under the rigid anchoring hypothesis they have theoretically interpreted the results and also considered other cases [13] where the transition might be of first order. They have also used the bend geometry [12] to demonstrate **E** induced biaxiality in nematics. Interestingly, they have observed [11, 12] modulated distortions under the influence of a stabilizing **H** above the bend threshold. A simple interpretation of the modulated structures has been given for a discontinuous transition [15]; conclusions in qualitative agreement with experiment have also been arrived at rigorously [16] with the small perturbations approach. A thorough study of the homogeneous transition should be helpful in developing a more complete theory of the modulated phase.

This may also shed light in another direction. If the bend transition is of second order a determination of the threshold and study of the deformation above threshold can enable estimation of K_3, K_1/K_3 and also certain other material parameters. Hence, a study of the homogeneous transition with a view to diminish or eliminate the discontinuity appears worth undertaking. It should also be interesting to find out how the nature of discontinuity is affected by changing the direction and magnitude of **H** and also by varying the initial director tilt at the sample planes. At this stage the following remarks may be made.

The occurrence of discontinuity in the bend transition can be appreciated because (i) ε_a is large so that the director field can experience a large electric torque; (ii) $\varepsilon_\parallel$ is large compared to $\varepsilon_\perp$; the director field aligning alone **E** is energetically favourable; (iii) **E** inside the sample is no longer parallel to the sample planes in the presence of director perturbations; **E** develops a component normal to the sample planes; the value of this component varies in space; (iv) the deformation occurring above the bend transition is large and hence splay dominant; as $K_1 < K_3$, this is energetically favourable.

While points (i) and (ii) hold in twist geometry, (iii) and (iv) appear to be non-existent. The deformation depends upon only one elastic constant K_2; it is also difficult to visualize physically how **E** inside the sample can get distorted.

Another aspect is the formation of domain walls [4]. In many situations, involving Freedericksz transitions, homogeneous distortions of opposite parity become equally energetic at threshold; above threshold these result in the formation of domains separated by domain walls. While the deformation away from the walls is predominantly homogeneous, the director distortion inside the walls defies a rigorous explanation by the continuum theory. There is no reason to exclude the formation of domains in a first order transition, nor can one rule out the possible effect domain formation might have on the width of hysteresis. It must be borne in mind that from the theoretical viewpoint [11–16] the distortion in any one domain is studied at points far away from the wall.

Lastly, the importance of the finiteness of the anchoring energy must not be lost sight of [17–19]. The rigid anchoring hypothesis is a convenient theoretical assumption which leads to close agreement with experiment in many situations. Still, under the action of **E** the combined effects of flexoelectricity and weak anchoring may be considerable [20].

In this communication an attempt is made to study some of the above aspects rigorously. In §2, the main differential equations governing splay-bend and twist deformations are set up; the case of twist is studied in some detail. The effects of varying the magnitude and direction of **H** and of varying the boundary tilt of director orientation form the subject of §3. Section 4 addresses itself to a brief discussion of the effects of weak anchoring compounded by flexoelectricity; §5 concludes the discussion.

2. Mathematical model, boundary conditions

In principle, one can start with the analytical formulation of [14]. Still, for reasons which become clear the problem is formulated *ab initio*. A nematic insulator is confined between two isotropic dielectric plates $z = \pm h$ and the entire sample is sandwiched between electrodes $x = \pm g/2$ between which a uniform potential difference V is maintained. By assuming that the nematic is an insulator we are ignoring the presence of free charges. We also assume that the lateral dimensions of the sample are large compared to the sample thickness so that we essentially consider regions of the sample which are far from the peripheries; thus, for instance, $g \gg 2h$. Inside the boundary plates the electric field and the dielectric induction are both parallel to x. Inside the nematic, however, owing to non-uniform director orientation, $\mathbf{E} = (E_x, E_y, E_z)$ in general with the components depending on z alone. This assumption is in keeping with the homogeneous nature of director deformation which is to be studied and also with the aspect ratio of the nematic sample. Maxwell's curl equation (curl $\mathbf{E} = 0$) [21] requires that E_x and E_y be constant inside the nematic while the divergence equation (div $\mathbf{D} = 0$ where $\mathbf{D}$ is the dielectric induction inside the nematic) leads to D_z = constant. At the boundary between two dielectrics the tangential components of **E** and the normal component of **D** must be continuous [21]. Hence

$$E_y = 0; \quad D_z = 0 \tag{2.1}$$

inside the nematic. At this stage it is instructive to treat two separate cases; cgs units are used throughout.

2.1. *Splay–bend distortions*

The deformed director field

$$\mathbf{n} = (S_\theta, 0, C_\theta); \quad S_\theta = \sin\theta; \quad C_\theta = \cos\theta; \quad \theta = \theta(z) \tag{2.2}$$

is assumed to lie in the xz plane under the influence of a magnetic field

$$\mathbf{H} = (HS_\psi, 0, HC_\psi). \tag{2.3}$$

In the nematic

$$D_i = \varepsilon_{ij}E_j + 4\pi P_i; \quad \varepsilon_{ij} = \varepsilon_\perp \varepsilon_{ij} + \varepsilon_a n_i n_j; \quad P_i = e_1 n_i(n_{k,k}) + \varepsilon_3 n_k n_{i,k}, \tag{2.4}$$

where repeated indices are summed, ε_{ij} is the dielectric tensor, P_i the flexoelectric polarization [8], e_1, e_3 the flexoelectric coefficients and a subscript comma denotes partial differentiation. The condition $D_z = 0$ leads to

$$E_z = [4\pi(e_1 + e_3)\theta_{,z} - \varepsilon_a E_x]S_\theta C_\theta/(\varepsilon_\perp + \varepsilon_a C_\theta^2) \tag{2.5}$$

showing that **E** develops a z component which varies with z and whose magnitude depends, in principle, on the flexoelectric constants. The bulk free energy density

$$\left.\begin{aligned} W &= K_1(\operatorname{div}\mathbf{n})^2/2 + K_2(\mathbf{n}\cdot\operatorname{curl}\mathbf{n})^2/2 \\ &\quad + K_3(\mathbf{n}\cdot\operatorname{grad}\mathbf{n})^2/2 - D_iE_i/8\pi - \chi_a(\mathbf{H}\cdot\mathbf{n})^2/2 \\ &= f_1(\theta)\theta_{,z}^2/2 - \chi_a H^2\{\cos^2(\psi-\theta)\}/2 + f_2(\theta)E_x^2/2 + f_3(\theta)E_x\theta_{,z}; \\ f_1(\theta) &= K_1S_\theta^2 + K_3C_\theta^2;\quad f_4(\theta) = \varepsilon_\perp + \varepsilon_a C_\theta^2;\quad f_2(\theta) = -\varepsilon_\perp\varepsilon_\parallel/4\pi f_4(\theta); \\ f_3(\theta) &= (e_1S_\theta^2\varepsilon_\perp - e_3C_\theta^2\varepsilon_\parallel)/2f_4(\theta) \end{aligned}\right\} \quad (2.6)$$

leads to the equation of equilibrium

$$f_1(\theta)\theta_{,zz} + (\partial f_1/\partial\theta)\theta_{,z}^2/2 + \chi_a H^2\{\sin(2\psi - 2\theta)\}/2 - (\partial f_2/\partial\theta)E_x^2/2 = 0. \quad (2.7)$$

Rigid anchoring requires that

$$\theta(z=-h) = \theta_1;\quad \theta(z=+h) = \theta_2. \quad (2.8)$$

If, on the other hand, the surface energy density

$$W_S = B_\theta[\sin^2\{\theta(z=+h)-\theta_2\}]/2 + \bar{B}_\theta[\sin^2\{\theta(z=-h)-\theta_1\}]/2, \quad (2.9)$$

where B_θ, $\bar{B}_\theta$ are the splay anchoring strengths at $z = +h, -h$, respectively [17–19], the boundary conditions take the form

$$\left.\begin{aligned} (\partial W/\partial\theta_{,z}) + B_\theta\{\sin(2\theta - 2\theta_2)\}/2 = 0 \quad \text{at} \quad z = +h; \\ (\partial W/\partial\theta_{,z}) - \bar{B}_\theta\{\sin(2\theta - 2\theta_1)\}/2 = 0 \quad \text{at} \quad z = -h. \end{aligned}\right\} \quad (2.10)$$

Evidently, flexoelectricity determines the bulk free energy but does not influence the equations of equilibrium (it acts like a surface term); still, if the director is not rigidly anchored at the boundaries, flexoelectricity can influence the director orientation by affecting the surface torques. For average anchoring strength B, elastic constant K and flexoelectric coefficient e, anchoring can be considered to be rigid if

$$Bh/K \gg 1;\quad B \gg eE_x. \quad (2.11)$$

Considering that $K \sim 10^{-6}$ dyne, $h \sim 0{\cdot}01$ cm, $B \sim 10^{-2}$ erg cm^{-2} [19], $e \sim 10^{-3}$ cgs [10], $E_x \sim 0{\cdot}1$ cgs (40 V across a gap of 0·33 cm [11]), equation (2.11) appears to apply. For the moment, therefore, we use equation (2.8) and come back to equation (2.10) later. A point to be noted is that it is not just ε_a which figures in the torque equations (2.7); the individual εs also enter the picture in a non-trivial way due to distortion of $\mathbf{E}$.

2.2. *Twist deformations*

If $\mathbf{n}_0$ lies in the xy plane prior to the application of $\mathbf{E}$, it is natural to seek solutions for the distorted orientation as

$$\mathbf{n} = (S_\phi, C_\phi, 0);\quad \phi = \phi(z). \quad (2.12)$$

As $D_z = \varepsilon_\perp E_z$, equation (2.1) implies that $E_z = 0$; *the electric field is not distorted* inside the sample. With $\mathbf{H} = (HS_\alpha, HC_\alpha, 0)$, the free energy density

$$W' = K_2\phi_{,z}^2/2 - (\varepsilon_\parallel S_\phi^2 + \varepsilon_\perp C_\phi^2)E_x^2/8\pi - \chi_a H^2\{\cos^2(\alpha - \phi)\}/2 \quad (2.13)$$

leads to the equation of equilibrium

$$K_2\phi_{,zz} + (\varepsilon_a E_x^2/4\pi)S_\phi C_\phi + \chi_a H^2\{\sin(2\alpha - 2\phi)\}/2 = 0 \quad (2.14)$$

to be solved with the rigid anchoring boundary conditions

$$\phi(z = -h) = \phi_1; \quad \phi(z = +h) = \phi_2. \tag{2.15}$$

If the anchoring is not rigid, the surface energy density [18, 19]

$$W' = B_\phi[\sin^2\{\phi(z = h) - \phi_2\}]/2 + \bar{B}_\phi[\sin^2\{\phi(z = -h) - \phi_1\}]/2 \tag{2.16}$$

determines the boundary conditions to be

$$\left.\begin{aligned} K_2\phi_{,z} + B_\phi\{\sin(2\phi - 2\phi_2)\}/2 &= 0 \quad \text{at} \quad z = +h; \\ K_2\phi_{,z} - \bar{B}_\phi\{\sin(2\phi - 2\phi_1)\}/2 &= 0 \quad \text{at} \quad z = -h \end{aligned}\right\} \tag{2.17}$$

where B_ϕ, $\bar{B}_\phi$ are the twist anchoring strengths. Due to the absence of splay and bend, flexoelectricity does not appear. The electric torque is determined by ε_a; the dielectric constants do not figure individually in equation (2.14). For the sake of definiteness consider rigid anchoring, equation (2.15), with $\phi_1 = \phi_2 = 0$ so that $\mathbf{n}_0$ is along y. Let $\mathbf{H}$ be stabilizing with $\alpha = 0$. Assuming that ϕ takes the extremum ϕ_m at $z = 0$, equation (2.14) can be integrated in one domain of deformation to yield ϕ_m as a function of H and E_x via the well-known elliptical integral formulation [4]. The limit $\phi_m \to 0$ leads to the (second order transition) threshold E_{xT} given by

$$(\varepsilon_a E_{xT}^2 h^2/4\pi K_2) - (\chi_a h^2 H^2/K_2) = \pi^2/4. \tag{2.18}$$

Exact calculations show that as E_x is decreased to E_{xT}, $\phi_m \to 0$; theoretically, the twist Freedericksz transition is found to be of second order even in the presence of a stabilizing $\mathbf{H}$. For $\mathbf{H}$ acting along x ($\alpha = \pi/2$) the sign of $\chi_a H^2$ is reversed in the expressions; equation (2.8) will then be meaningful only for H less than the twist Freedericksz value $(\pi/2h)(K_2/\chi_a)^{1/2}$. As the twist transition is of second order and the bend transition of first order we can envisage the possibility (theoretically) that there might exist a cut off tilt of $\mathbf{n}_0$ (away from the homeotropic and in a plane normal to $\mathbf{E}$) beyond which the transition becomes continuous. A rigorous treatment of this case would involve both orientational degrees of freedom and will be considered separately.

Interestingly it has been stated [11] that $\mathbf{E}$ induced twist transition is discontinuous and values of the hysteresis width have been arrived at theoretically (although details regarding calculations and experimental observations of hysteresis have not been presented). The theroetical conclusions arrived at *ab initio* in this section are in contradiction with those of [11]. Taking as correct the conclusions of this section and also the experimental observations of [11] it may be worth critically examining the mathematical model to suggest some way of bridging the gap between theory and experiment.

It must be remembered that in the experiment [11] the length scales along z, x and y are in the ratio 0·5 mm: 3·3 mm so that the aspect ratios of the sample can be written as 1 : 6·6 : 60. Although along y the lateral dimension of the sample is large compared to $2h$, along x it is not; i.e. $g = 6{\cdot}6\ (2h)$. Hence in a more complete description the boundary conditions at $x = \pm g/2$ must also be considered in addition to the boundary conditions at $z = \pm h$. Let us note that the interface at $z = \pm h$ is between two dielectrics while that at $x = \pm g/2$ is between a dielectric (nematic) and a conductor (the electrodes). It becomes immediately clear that a simple mathematical model (similar to the one developed here and in [11]) will lead to mathematical contradictions if attempts are made to impose both sets of boundary conditions on quantities which are assumed to depend on z alone. To study properly the twist transition in a realistic situation where the electrode gap is not large compared to sample thickness it may be necessary to assume that all relevant quantities depend on more than one coordinate; it may then be possible to impose both sets of boundary conditions not only on the electric field components but also on the director tilt angle(s).

Thus it may be concluded that when $g \gg 2h$, the twist transition may be continuous and when $g \sim 2h$ the transition may become discontinuous. If this is indeed the case it may be possible to find some critical electrode gap g_c such that for $g > g_c$ the twist transition is continuous and for $g < g_c$ the transition is of first order. This may be capable of being verified experimentally.

3. Results for splay–bend distortions

In addition to rigid anchoring at the sample boundaries we also assume that the director is symmetrically oriented at the sample planes; $\theta_1 = \theta_2$. Calculations have been carried out for the parameter set given below (these give a close representation for 5CB [16])

$$(K_1, K_2, K_3) = (3{\cdot}5, 2{\cdot}0, 4{\cdot}2) \times 10^{-7}\,\text{dyne}; \quad \varepsilon_\parallel = 18{\cdot}8,\ \varepsilon_\perp = 8{\cdot}2\,\text{esu};$$
$$\chi_a = 0{\cdot}8 \times 10^{-7}\,\text{emu};\ h = 0{\cdot}01\,\text{cm}. \tag{3.1}$$

The qualitative nature of the discussion is unaffected by expressing results for threshold in terms of electric field ratios, hence the electrode gap width is not relevant. Angles are expressed in radians. Assuming that θ, equation (2.2), attains an extremum at the sample centre ($\theta = \theta_m$ at $z = 0$) one integrates equation (2.7) to get

$$\left.\begin{aligned}(d\theta/dz)^2 &= f_5(\theta)/f_1(\theta);\\ f_5(\theta) &= \chi_a H^2[\sin^2(\psi - \theta) - \sin^2(\psi - \theta_m)]\\ &\quad + (\varepsilon_a \varepsilon_\perp \varepsilon_\parallel E_x^2/4\pi)(\sin^2\theta_m - \sin^2\theta)/f_4(\theta)f_4(\theta_m).\end{aligned}\right\} \tag{3.2}$$

Choosing the domain in which θ increases from θ_1 to θ_m, equation (3.2) is further integrated to yield

$$h = \int_{\theta_1}^{\theta_m} [f_1(\theta)/f_5(\theta)]^{1/2}\, d\theta, \tag{3.3}$$

from which θ_m can be calculated iteratively for given values of H, E_x and other parameters. As the integral, equation (3.3), is not well behaved at all values of θ_1 and θ_m, the transformation

$$\sin\mu = \sin(\theta_m - \theta_1)\sin\lambda; \quad \mu = \theta - \theta_1 \tag{3.4}$$

is utilized to write the integral in terms of λ. The integral can be evaluated by gaussian qaudrature [22]. The total free energy F can be similarly calculated from

$$F = 2\int_{\theta_1}^{\theta_m} W(dz/d\theta)\, d\theta, \tag{3.5}$$

after using equation (3.4); the factor 2 appears in equation (3.5) due to the symmetric nature of θ. For a domain in which θ decreases to θ_m, one simply changes the sign of the integrals in equation (3.3) and (3.5). The calculation of F helps determine whether a particular branch of the solution is physically realistic or not. Needless to say, equation (3.3) and (3.5) reduce to the corresponding expressions of [11] when $\theta_1 = 0$ (homeotropic initial orientation).

The flexoelectric term $f_3(\theta)E_x\theta_{,z}$ can be dropped from W for calculation of F in the present case as it vanishes identically when integrated over the sample thickness. Hence, under rigid anchoring the flexoelectric term does not determine F when the initial orientation of the director is symmetric. On the other hand for asymmetric anchoring, i.e. $\theta_1 \neq \theta_2$, (2.8), one can expect that flexoelectricity might determine F as θ now becomes an asymmetric function of z. In addition, this term is linear in E_x; this means that its contribution to F can even reverse sign when E_x is reversed. As the calculation of deformation is not straightforward for asymmetric anchoring, this case will be considered separately. We shall now take up some special cases of symmetric anchoring.

3.1. *Bend Freedericksz transition*

$\theta_1 = \theta_2 = 0$. For a stabilizing (destabilizing) **H** along z (along x) $\psi = 0$ ($\psi = \pi/2$). In equation (3.4) $\mu = \theta$ itself. Taking the limit $\theta_m \to \theta_1 = 0$ in equation (3.3), one recovers the well-known expression

$$\left.\begin{aligned} E_{xc} &= [\{(K_3\pi^2/4h^2) + \delta\chi_a H^2\}(4\pi\varepsilon_\parallel/\varepsilon_a\varepsilon_\perp)]^{1/2}; \\ \delta &= +1 \quad \text{if} \quad \chi_a > 0, \quad \mathbf{H}_S = (0,0,H); \\ \delta &= -1 \quad \text{if} \quad \chi_a < 0, \quad \mathbf{H}_D = (0,0,H) \\ &\qquad \text{or if} \quad \chi_a > 0, \quad \mathbf{H}_D = (H,0,0); \end{aligned}\right\} \tag{3.6}$$

for the (second order) Freedericksz threshold; $\mathbf{H}_S$, $\mathbf{H}_D$ correspond to stabilizing and destabilizing magnetic fields, respectively. Interestingly, *reversing the sign of* χ_a *and applying* **H** *along* z *is formally identical to the case where* χ_a *remains unchanged but* **H** *acts along* x.

Figure 1 depicts the variation of the order parameter θ_m as a function of $R = E_x/E_{xc}$ for different parameters. One domain ($\mathscr{D}_1$) has been chosen in which θ increases to θ_m the other domain ($\mathscr{D}_2$) can be represented by simply changing the sign of θ. The results of figure 1 (*a*) are in qualitative agreement with the findings of [11]. At $R = 1$, θ_m takes large values for $H = 0$ (curve 3). When R is decreased, θ_m diminishes and becomes double valued in the range $R_m < R < 1$; for $R < R_m$, non-trivial values of θ_m cannot be found, (i.e. there is no distortion) so that $1 - R_m$ gives a measure of the transition width. In the region of bistability the lower deformation state has a higher F and is therefore represented by a dashed curve. For a material such as 5CB ($\chi_a > 0$), a stabilizing $\mathbf{H}_S$ enhances the transition width and also the extent of discontinuity while a destabilizing $\mathbf{H}_D$ has the opposite effect. This is easy to grasp intuitively by noting, (3.6), that $\mathbf{H}_S$ ($\mathbf{H}_D$) enhances (diminishes) the 'effective bend elastic constant' relative to K_1. Results for $\mathbf{H}_D$ are obtained by simply reversing the sign of $\chi_a H^2$ in equation (3.3) and (3.5). A sufficiently strong $\mathbf{H}_D$ (of strength less than the magnetic bend threshold) can even remove the discontinuity in the transition. In [11] the destabilizing action of **H** acting along z on a negative χ_a nematic is considered. It should be clear from figure 1 (*a*) that formally identical results are obtained for a χ_a positive nematic subjected to **H** acting along x. An advantage of making the bend transition continuous is that the threshold is then accurately described by equation (3.6), in addition, by using equations (3.3) to study the deformation above threshold it may be possible to estimate certain ratios of material parameters as has been done for other field orientations [5]. Considering the rather different behaviour of **H** acting along z and x, one realizes that by applying **H** at intermediate angles it should be possible to influence bistability in more interesting ways; this will be seen later.

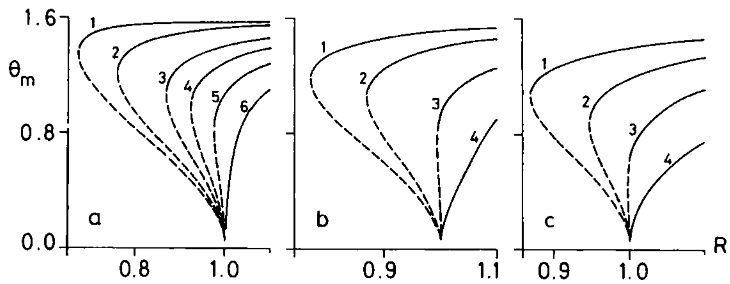

Figure 1. Plots of θ_m (radians) versus $R = E_x/E_{xc}$ in the bend geometry. θ_m is the (extremum) angle of deviation away from the homeotropic at the sample centre. E_x is the component of **E** acting along the sample planes ($z = \pm h$) and E_{xc} equation (3.6), is the electric bend Freedericksz threshold for a continuous transition if one were possible. The parameter set, equation (3.1), is used. In the region of bistability the dashed parts of the curves correspond to a distortion having higher total free energy F. (*a*) shows the effect of **H** on the nature of the transition. A stabilizing $H =$ (1) 720 (2) 360 emu makes the transition more discontinuous and also increases the width. Curve (3) is for $H = 0$. A destabilizing **H** along x can decrease the magnitude of the jump at the transition or even make the transition continuous; $H =$ (4) 180 (5) 235 (6) 270 emu. Curves identical to curves (4)–(6) would be obtained for a material with $\chi_a = -0{\cdot}8 \times 10^{-7}$ emu if **H** were applied along z. (*b*) Effect of increasing K_1 while keeping other parameters as in equation (3.1). $K_1 =$ (1) 1·75 (2) 3·5 (3) 7·0 (4) 10·5 $\times 10^{-7}$ dyne. (*c*) Effect of diminishing $\varepsilon_\parallel$; other parameters as in equation (3.1). $\varepsilon_\parallel =$ (1) 18·8 (2) 15·0 (3) 12·0 (4) 9·0 esu. Increase of K_1 or diminution of $\varepsilon_\parallel$ is detrimental to the occurrence of a first order bend transition. The effects of decreasing K_3 or enhancing $\varepsilon_\perp$ are similar but diagrams have not been shown. The distortion in the other domain can be found by simply changing the sign of θ_m in the diagrams. The conclusions of (*a*) are in good agreement with those of [11].

The effects of varying some of the other parameters on the nature of the transition are shown in figures 1 (*b*), (*c*). While one parameter out of the set, equation (3.1), is varied, the others are kept constant at their original values. It is seen that sufficient enhancement of K_1 relative to K_3 (or diminution of $\varepsilon_\parallel$ with respect to $\varepsilon_\perp$) can make the bend transition continuous. The effects of diminishing K_3 or, enhancing $\varepsilon_\perp$ are similar and are not shown.

3.2. *Homeotropic initial orientation*; ***H*** *at angle ψ with the z axis*; *variation of E_x*

One value of $H(= 360$ emu) is chosen. **H** is applied at different angles ψ and results for $\psi = 0$ are given for comparison. The destabilizing field is measured in units of $R = E_x/E_{xc}$ ($\psi = 0$) where the denominator is obtained from equation (3.6) for a purely stabilizing **H**. A little thought makes it clear that the nature of distortion will depend on the sign and magnitude of ψ, the kind of domain and also on the order in which **E** and **H** are applied. The results are summarized in figure 2.

Consider the effect of **H** being applied first on the homeotropically aligned sample. This produces a monodomain deformation $\mathscr{D}_1$ (or $\mathscr{D}_2$) for $\psi > 0$ (or $\psi < 0$) with $|\theta_m| \lesssim |\psi|$. Now E is applied and E_x(or R) increased from a low value. The distortion in the sample appreciates (figure 2 (*c*) or (*b*)) before attaining saturation. When ψ is sufficiently different from zero (curves 3) the enhancement of deformation is practically continuous. When ψ is close to zero, however, the distortion shows a discontinuous jump (curves 2) before attaining saturation. At the end what remains is the monodomain $\mathscr{D}_1$ (or $\mathscr{D}_2$) with which we started.

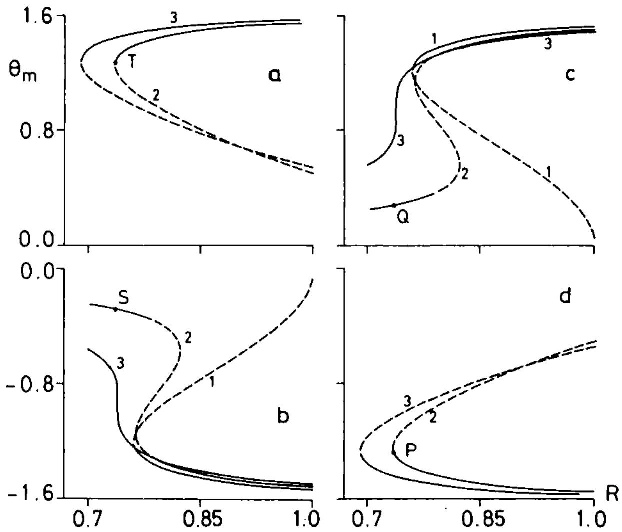

Figure 2. Variation of θ_m as a function of R for $H = 360$ emu applied at different angles ψ with the z axis. Initial alignment is homeotropic (along z). $R = E_x/E_{xc}$ ($\psi = 0$) where the denominator is calculated from equation (3.6) for a stabilizing $H = 360$ emu. In (c) and (d) ψ = (1) 0·0 (2) 0·2 (3) 0·4 radian. In (a) and (b) the sign of ψ is reversed. In (b) and (c) $\theta_m \to \psi$ as $R \to 0$; parts of the curves in this region have not been shown. (a), (c) and (b), (d) represent domains $\mathscr{D}_1$, $\mathscr{D}_2$ respectively. As explained in §3.2, the order in which **H** and **E** are applied on the sample can be important for $\psi \neq 0$. When **H** is oblique, depending upon the sign of ψ, one set of domains may get annihilated; it may be possible to convert irreversibly a multidomain deformation to a monodomain type.

Suppose initially, $H = 0$ and E_x is increased from a low value. When the bend threshold is exceeded, domains $\mathscr{D}_1$, $\mathscr{D}_2$ will form. With E_x at a sufficiently elevated value **H** is applied at angle ψ with the z axis and R is now diminished. When R is sufficiently high, the deformation does not change appreciably. To fix ideas left $\psi > 0$. In domain $\mathscr{D}_1$ the distortion decreases continuously or otherwise as R is diminished (figure 2(c)) as explained in the previous paragraph; when $R \to 0$, $\theta_m \to \psi$. On the other hand, the variation of θ_m in $\mathscr{D}_2$ is rather different. Initially, as R is decreased $|\theta_m|$ diminishes (figure 2(d)). When R is less than a lower limit R_L, θ_m changes sign discontinuously from a large negative value (for instance, from point P of figure 2(d), curve 2 for $\psi = 0{\cdot}2$) to a positive value (point Q of figure 2(c), curve 2 for $\psi = 0{\cdot}2$). This is clearly because at small E_x, **H** dictates the orientation; as ψ is positive, θ_m also must be positive. When R is further reduced to zero, $\theta_m \to \psi$. Effectively, therefore, when $R < R_L$, the deformation in the original domain $\mathscr{D}_2$ changes discontinuously to that of the $\mathscr{D}_1$ kind; but in the original $\mathscr{D}_1$ domain, the distortion remains qualitatively the same. This suggests the possibility of applying an oblique **H** on a multidomain sample and changing the distortion to a monodomain kind by reducing the strength of the electric field. Needless to say, if we started with $\psi < 0$, we would similarly go over from $\mathscr{D}_1$ to $\mathscr{D}_2$ (the transition from T of figure 2(a) to S of figure 2(b) at $\psi = -0{\cdot}2$) and obtain a deformation of the $\mathscr{D}_2$ type. A point worth noting is that such a transition would be irreversible. For $R < R_L$ consider the transition P to Q (figures 2(d), (c)). Subsequent to the transition, if R were increased above R_L, θ_m would only take higher positive values as in figure 2(c); the distortion would never attain negative values of θ_m as in figure 2(d).

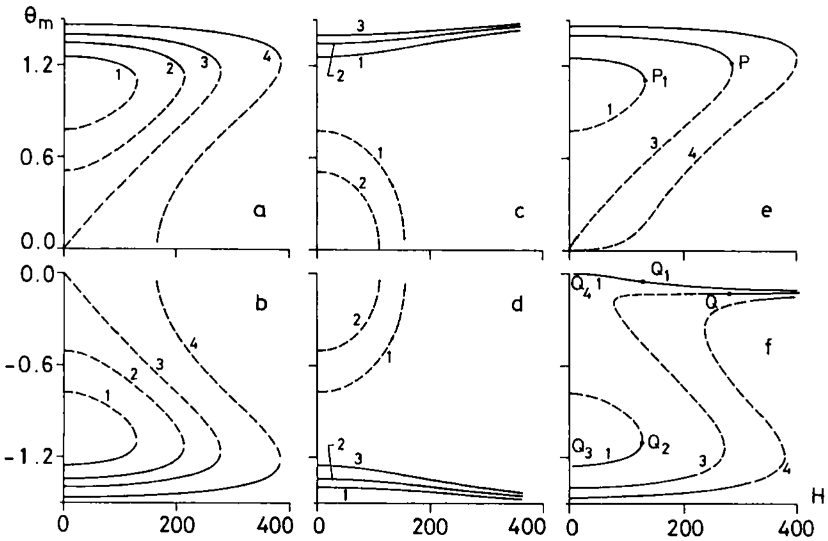

Figure 3. Plots of θ_m against H for different values of the tilt angle ψ and electric field measured in units of $R = E_x/E_{xc}$ ($H = 0$) where the denominator is calculated from equation (3.6) for zero magnetic field. Initial alignment is homeotropic. $\psi = 0$ in (*a*) and (*b*) (**H** is purely stabilizing); $\psi = \pi/2$ in (*c*) and (*d*) (**H** is along the sample planes; purely destabilizing); $\psi = -0{\cdot}1$ in (*e*) and (*f*) (**H** is applied at a small oblique angle with the z axis). Curves are drawn for $R =$ (1) 0·9 (2) 0·95 (3) 1·0 (4) 1·1. In (1) and (2) it is assumed that the bend threshold is exceeded and E_x subsequently diminished. It is also clear that while (*a*), (*c*) and (*e*) represent domain $\mathscr{D}_1$, (*b*), (*d*) and (*f*) represent domain $\mathscr{D}_2$. Figures 3 (*a*) and (*b*) show that a deformation discontinuously induced by **E** can be discontinuously quenched by a stabilizing **H**. When $\psi = \pi/2$, increase of H continuously enhances the distortion previously produced by E_x (figures 3 (*c*) and (*d*)). When $\psi = -0{\cdot}1$ (figures 3 (*e*) and (*f*)) enhancing H diminishes the deformation in $\mathscr{D}_2$ discontinuously while the distortion in $\mathscr{D}_1$ changes to the $\mathscr{D}_2$ type. For $R < 1$ (curves 1, figures 3 (*e*) and (*f*)) the order in which **H** and **E** are applied becomes important in determining how θ_m evolves (§3.3).

3.3. *Homeotropic initial alignment; H at angle ψ with z; variation of H*

Figure 3 shows plots of θ_m versus H at three values of $\psi = (0, \pi/2, -0{\cdot}1)$ for different values of $R = E_x/E_{xc}$ ($H = 0$) where the denominator is calculated from equation (3.6) for zero H. At $\psi = 0$ (figures 3 (*a*), (*b*)), $|\theta_m|$ diminishes when H is enhanced for a given R. When H exceeds an upper limit H_L, the deformation disappears discontinuously in both domains; the greater the R, the higher the corresponding H_L. Subsequently when H is diminished below H_L it is reasonable to expect that the distortion will reappear discontinuously accompanied, perhaps, by the reformation of domains. Thus, the deformation discontinuously produced by E can be suppressed discontinuously by applying a sufficiently strong stabilizing **H**; decrease of H might restore the distortion discontinuously. When $\psi = \pi/2$, $|\theta_m|$ increases continuously with H (figures 3 (*c*), (*d*)).

When $\psi = -0{\cdot}1$ (figures 3 (*e*), (*f*)) the effect of enhancing H can be rather different. It seems appropriate to consider the two cases $R \geqslant 1$ and $R < 1$ separately. Let $R = 1{\cdot}0$ or $1{\cdot}1$ (curves 3, 4). Consider the domain $\mathscr{D}_2$ (figure 3 (*f*)). When H is increased beyond an upper limit, $|\theta_m|$ decreases discontinuously to a lower value; on further enhancement of H, $\theta_m \to \psi$. If H is subsequently diminished continuously to zero, θ_m will vary reversibly as shown by curves 3, 4. In domain $\mathscr{D}_1$, on the other hand,

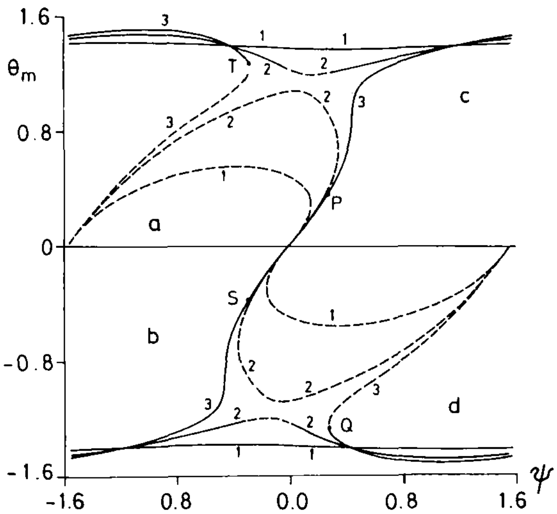

Figure 4. θ_m versus ψ for different H. Homeotropic initial alignment. Curves are drawn for H = (1) 120 (2) 270 (3) 330 emu. $R = E_x/E_{xc}$ ($H = 0$) = 1·0. (*a*), (*c*) and (*b*), (*d*) represent, respectively, domains $\mathscr{D}_1$, $\mathscr{D}_2$. The distortion produced by **E** is appreciably affected by changing ψ only if H is high enough. It is found (§3.4) that if H is large enough it may be possible to change a multidomain deformation into a monodomain deformation by varying the tilt of **H**.

θ_m initially decreases as H is increased (figure 3 (*e*), curves 3, 4). When H exceeds an upper limit, θ_m decreases in a discontinuous jump from a positive value to a negative value (for instance, at R = 1·0, the transition from point P, figure 3 (*e*) to point Q figure 3 (*f*)). This is clearly because H now dictates the sign of the distortion at the sample centre. Further enhancement of H will only take θ_m closer to ψ as in figure 3 (*f*). Thus enhancement of H at a small negative ψ may irreversibly annihilate domains $\mathscr{D}_1$ yielding a monodomain $\mathscr{D}_2$ deformation. In the same way if we were to start with ψ = +0·1 we would be left with a monodomain $\mathscr{D}_1$.

When R = 0·9 (curves 1, figures 3 (*e*), (*f*)) the transition from $\mathscr{D}_1$ to $\mathscr{D}_2$ is straightforward (from P_1, figure 3 (*e*) to Q_1 figure 3 (*f*)). However, the order of applying **E** and **H** may be important. Suppose H = 400 emu is applied at ψ = −0·1 and then **E** at R = 0·9. A monodomain $\mathscr{D}_2$ results with $\theta_m \approx \psi$. When H is decreased to zero, $|\theta_m|$ diminishes to zero along curve 1 (top of figure 3 (*f*)). Suppose, on the other hand **E** was first applied at $R > 1$ and R subsequently decreased to 0·9. Domains $\mathscr{D}_1$, $\mathscr{D}_2$ would form with $|\theta_m| \approx 1{\cdot}2$ (figures 3 (*e*), (*f*)). Now if H is enhanced, $|\theta_m|$ diminishes in both $\mathscr{D}_1$, $\mathscr{D}_2$. When H exceeds a limit, θ_m changes discontinuously in $\mathscr{D}_2$ to a value close to ψ (Q_2 to Q_1 in figure 3 (*f*)); interestingly, at about the same H the deformation in $\mathscr{D}_1$ would change to the $\mathscr{D}_2$ type. It must be remembered that if H were to be decreased now, $|\theta_m|$ would decrease to zero; θ_m would never attain values corresponding to the part Q_3 to Q_2 of curve 1 (calculation shows that the part Q_3 to Q_2 of curve 1 has a higher F than the part Q_4 to Q_1; still, the part Q_2 to Q_3 represents a metastable state which can exist only when **E** is applied before **H**; this is why Q_2 to Q_3 has not been represented by a dashed line).

3.4. *Homeotropic initial alignment*; H *at angle* ψ; *variation of* ψ

Figure 4 represents the variation of θ_m with ψ for one value of $R = E_x/E_{xc}$ ($H = 0$) = 1·0. First, **E** is applied and domains $\mathscr{D}_1$, $\mathscr{D}_2$ are formed. **H** is now applied at $\psi = \pi/2$ (or at $\psi = -\pi/2$) and rotated in the xz plane to the opposite extreme $\psi = -\pi/2$ (or $\psi = +\pi/2$). Noting, from figures 3 (*a*), (*b*) that $H_L \approx 270$ emu, three values of H (= 120, 270, 330 emu) are chosen. When H (= 120) is well below H_L, variation of ψ has very little effect on the distortion in $\mathscr{D}_1$ or $\mathscr{D}_2$.

When $H = 270$ (curve 2) the deformation changes in a more significant way. It is again convenient to study $\mathscr{D}_1$, $\mathscr{D}_2$ separately. In $\mathscr{D}_1$, θ_m diminishes continuously with ψ up to a limit ψ_c; for $\psi \lesssim \psi_c$, θ_m jumps to a lower value and then decreases continuously with ψ as $\psi \to 0$ (figure 4 (*c*)). Thus, at $\psi = 0$, the distortion in $\mathscr{D}_1$ should practically vanish. When ψ diminishes further to negative values, $|\theta_m|$ increases; the deformation changes to the $\mathscr{D}_2$ type (figure 4 (*b*)). At $\psi = -\psi_c$, $|\theta_m|$ increases with a jump; for $\psi < -\psi_c$ the variation of θ_m with ψ again becomes continuous. It must be noted that as the deformation represented by curve 2, figure 4 (*a*), is more energetic than that corresponding to curve 2 of figure 4 (*b*), there is no possibility of θ_m assuming positive values once ψ has changed sign; effectively the $\mathscr{D}_1$ type distortion changes continuously to the $\mathscr{D}_2$ type when ψ changes sign.

Consider now how the deformation varies in $\mathscr{D}_2$. When ψ is decreased from $\pi/2$, $|\theta_m|$ diminishes continuously; as $\psi \to 0$, θ_m changes discontinuously to zero so that the distortion in $\mathscr{D}_2$ gets extinguished at about the same point when the deformation in $\mathscr{D}_1$ vanishes (figure 4 (*d*)). When ψ changes sign, $|\theta_m|$ varies as already described (figure 4 (*b*)). Thus, despite jumps occurring in θ_m at $\pm\psi_c$, the nature of deformation changes from the multidomain type to the $\mathscr{D}_2$ type when ψ is varied from $+\pi/2$ to $-\pi/2$. A similar reasoning shows that if one started with a multidomain distortion, changing ψ from $-\pi/2$ to $+\pi/2$ would result in a monodomain $\mathscr{D}_1$ when ψ changes sign.

When H (= 330) is greater than H_L, the change in deformation occurs in a totally different way. When ψ is decreased from $\pi/2$, θ_m in $\mathscr{D}_1$ continuously diminishes; in $\mathscr{D}_2$, $|\theta_m|$ also decreases up to a point. Then, the distortion in $\mathscr{D}_2$ changes discontinuously to the $\mathscr{D}_1$ type (transition from Q, curve 3, figure 4 (*d*) to P, figure 4 (*c*)); at this stage one expects that the sample becomes a monodomain $\mathscr{D}_1$. When ψ becomes zero, $\theta_m \to 0$; when ψ changes sign so does θ_m with the deformation changing to the $\mathscr{D}_2$ type; for further variation of ψ, θ_m varies as in figure 4 (*b*). For both $H = 270$ and 330, the changes in the nature of domains are irreversible; once a monodomain deformation is formed there is no way by which it can break up into a multidomain one as long as **H** is acting on the sample.

3.5. *Tilted initial alignment*; $\theta_1 \neq 0$; $\mathbf{H} = 0$

In this subsection results are presented on the effects of the initial director tilt on **E** induced deformation. The tilt of $\mathbf{n}_0$ with respect to z is θ_1 in the xz plane; rigid anchoring is assumed. When $\theta_1 \neq 0$, a Freedericksz threshold no longer exists for orientational changes from the uniformly aligned state. Still, noting the results from the previous sections discontinuous changes can be expected to take place between states of low and high deformation. When $\theta_1 > 0$ (or $\theta_1 < 0$) under the action of **E**, a monodomain $\mathscr{D}_1$ (or $\mathscr{D}_2$) will result. The deformation is symmetric so that θ assumes an extremum θ_m at the sample centre and θ_m can be determined from equation (3.3) to (3.5).

Figures 5 (*a*), (*b*) show the variation of θ_m with θ_1 for different values of $R = E_x/E_{xc}$ ($H = 0$) in $\mathscr{D}_1$, $\mathscr{D}_2$. In general $|\theta_m|$ is large when θ_1 is sufficiently different from zero. The value of R determines the nature of the variation of $|\theta_m|$ with θ_1 as θ_1

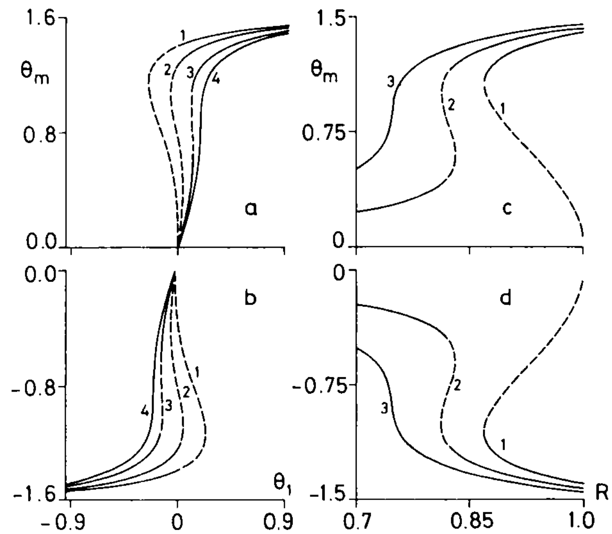

Figure 5. Variation of θ_m as a function of θ_l and $R = E_x/E_{xc}$ ($H = 0$) in the absence of **H**. θ_l is the angle made by the initial uniform director orientation with z axis (in the xz plane). (*a*) and (*b*) θ_m versus θ_l for R = (1) 1·0 (2) 0·9 (3) 0·8 (4) 0·75 in domains $\mathscr{D}_1$, $\mathscr{D}_2$. When E_x is high enough θ_m changes sign discontinuously as θ_l changes sign. (*c*) and (*d*) plots of θ_m as a function of R for different θ_l. In (*c*) (domain $\mathscr{D}_1$) θ_l = (1) 0·0 (2) 0·1 (3) 0·2; in (*d*) (domain $\mathscr{D}_2$) the sign of θ_l is reversed. It must be remembered that when R is small, $\theta_m \to \theta_l$; this portion of the curves is not shown fully. The deformation changes continuously with E_x when the initial director tilt is sufficiently away from the homeotropic (§3.5).

approaches zero. For small enough R (=0·75, curves 4), $|\theta_m|$ decreases continuously to zero as $\theta_l \to 0$. When R is slightly higher (=0·8, curves 3) $|\theta_m|$ initially decreases continuously as $|\theta_l|$ is diminished; when θ_l changes further, $|\theta_m|$ decreases discontinuously before approaching zero. For both the above R values, θ_m changes sign continuously as θ_l changes sign. When R takes still higher values (=0·9, 1·0, curves 2, 1) $|\theta_m|$ decreases continuously as $\theta_l \to 0$. When θ_l changes sign, however, θ_m changes sign discontinuously.

Figures 5(*c*), (*d*) depict plots of θ_m as a function of R for different θ_l. Figures 5(*c*), (*d*) represent domains $\mathscr{D}_1$ ($\mathscr{D}_2$) occurring for θ_l = 0·0, 0·1, 0·2 (0·0, −0·1, −0·2). For $\theta_l = 0$, the change of distortion occurs as depicted in figure 1(*a*). When θ_l is slightly different from zero $|\theta_m|$ initially decreases continuously with R; when R decreases further, $|\theta_m|$ diminishes discontinuously; when $R \to 0$, $\theta_m \to \theta_l$. The jump can be understood to occur when control over the distortion is exerted more by the substrate than by **E**. When θ_l is sufficiently removed from zero, θ_m changes continuously as R is varied.

In principle it should also be possible to study orientational changes occurring under the competitive actions of **E** and **H** in the present case. These results will be presented separately because a detailed exposition falls outside the scope of this work; in addition it must be remembered that application of **H** at certain angles, even in the absence of **E**, might lead to discontinuous orientational changes (see, for instance, [23]).

The above discussion has dealt with only symmetric boundary conditions. It is possible to think of two other cases. One is the asymmetric case where, equation (2.8), the director is anchored at different tilts at the two boundaries. This can be regarded as a generalization of the hybrid alignment [10]. As θ becomes asymmetric, $\theta\,(z = 0)$ is not an extremum. The flexoelectric term, which is linear in E_x, can determine F; this may assume importance in deciding which is the most stable among multiple solutions.

The other case is that of antisymmetric director anchoring (also known as the reversely pretilted case; see, for instance, [24]); $\theta_2 = -\theta_1$. Here again flexoelectricity may figure. The extremum of θ will again not occur at the sample centre. Other problems also arise because of the non-polarity of the director field which leads to formation of defects separating different kinds of distortions. As the computation of deformation is not straightforward in the above two cases, these will be presented separately. Intuitively, however, it can be seen that an asymmetric initial orientation might suppress the occurrence of discontinuous changes in θ; for instance when $\theta_1 = 0$ at one surface and θ_2 is sufficiently large one can expect distortion to change continuously when R is increased from a low value.

4. Homeotropic initial alignment; weak director anchoring at the sample planes

This topic actually falls outside the scope of this communication as it is difficult to present results in detail. Still, keeping in mind the experiment [12] it seems appropriate to deal briefly with this topic. So far the bend Freedericksz transition has been assumed to occur discontinuously from the state of uniform alignment ($\theta = 0$) to the deformed state when E_x exceeds a critical value, the orientation at the boundaries is assumed not to change whatever the E_x. In the experiment the transition is detected optically by measuring the intensity of light transmitted by the sample placed between crossed polarizers. If the rigid anchoring hypothesis picture is strictly valid the sample must transmit light at or above threshold and not below threshold. Interestingly it appears [12] that as E_x is slowly increased from zero the sample starts transmitting more and more light. When E_x exceeds the threshold the transmitted intensity shows a sudden jump. This phenomenon has been interpreted [12] by analysing thermal fluctuations of the director field. It is found that fluctuations build up more along x than along other directions, owing to the action of E; the resultant biaxiality causes the initial transmittance of light. Theoretical calculations are in good agreement with experiment [12]. Significantly, a strong stabilizing $\mathbf{H}$ ($H = 3300$ emu, $2h = 0{\cdot}05$ cm) has been used in the experiment; in addition $\mathbf{E}$ is an alternating field so that flexoelectric effects can be expected to be minimized.

From a purely phenomenological viewpoint the gradual increase of transmittance with E_x can also be regarded as being caused by a slow build up of director deformation occurring without a threshold. It will now be shown that such a situation can indeed arise due to flexoelectric intervention when director anchoring at the sample planes is not ideally rigid and $\mathbf{E}$ is a static field. To find out whether this has any bearing on the interpretation of the above experiment, we consider equations (2.7) and (2.10) in the limit of small deformations. Linearizing with respect to θ and putting $\theta_1 = \theta_2 = 0$ in equation (2.10) (assuming that the easy axis at both the plates is homeotropic) and including the action of a stabilizing $\mathbf{H}$ acting along z one obtains

$$\left.\begin{aligned} &d^2\theta/d\xi^2 + q^2\theta = 0;\quad \xi = z/h;\quad q^2 = [(\varepsilon_\perp \varepsilon_a E_x^2/4\pi\varepsilon_\parallel) - (\chi_a H^2](h^2/K_3);\\ &[d\theta/d\xi + \sigma_e E_x + \sigma_\theta \theta](\xi = +1) = 0\\ &\quad = [d\theta/d\xi + \sigma_e E_x - \bar{\sigma}_\theta \theta](\xi = -1);\\ &\sigma_e = -e_3 h/2K_3;\quad \sigma_\theta = B_\theta h/K_3;\quad \bar{\sigma}_\theta = \bar{B}_\theta h/K_3. \end{aligned}\right\}\qquad (4.1)$$

If we were to ignore flexoelectricity in equation (4.1) we would find a second order transition threshold depending on σ_θ, $\bar{\sigma}_\theta$. If we could similarly solve equations (2.7) and (2.10) ignoring flexoelectricity we could study the non-linear deformation above threshold and our results would reduce to those of §3 in the limit, equation (2.11). The presence of flexoelectricity makes such an exercise not very fruitful. As can be seen, equation (4.1), $e_3 E_x$ provides a driving term which can give rise to θ for any $E_x \neq 0$ without threshold. In addition, if the anchoring at the sample planes is not ideally rigid, $\theta(z = +h, -h)$ can also change.

There exist three possibilities for q^2. At low E_x and large H, $q^2 < 0$. As E_x is increased q^2 crosses zero and becomes positive. For $q^2 > 0$ (which should be valid close to the transition)

$$\left.\begin{aligned}
\theta &= \theta_s \sin q\xi + \theta_c \cos q\xi; \quad \theta_s = \gamma_1(\delta_2 - \delta_1)/\Delta; \quad \theta_c = \gamma_1(\beta_1 - \beta_2)/\Delta;\\
\Delta &= \delta_2\beta_1 - \delta_1\beta_2; \quad \gamma_1 = -\sigma_e E_x;\\
\delta_1 &= \sigma_\theta \cos q - q\sin q; \quad \delta_2 = -\bar{\sigma}_\theta \cos q + q \sin q;\\
\beta_1 &= \sigma_\theta \sin q + q\cos q; \quad \beta_2 = \bar{\sigma}_\theta \sin q + q\cos q.
\end{aligned}\right\} \quad (4.2)$$

Expressions for $q^2 < 0$ can be written by making q imaginary; for $q = 0$, one takes the limit $q \to 0$. It is seen that when the anchoring strengths at the two substrates are unequal ($\sigma_\theta \neq \bar{\sigma}_\theta$) θ is asymmetrical. As the amplitude of θ is proportional to E_x, θ changes sign with E_x. For equal anchoring strengths $\theta_c = 0$ and θ is purely antisymmetric. It is only in the small distortion limit that θ is determined by e_3 alone; in principle, as seen from equations (2.7) and (2.10), e_1 should also enter the picture, especially when the deformation is large. As seen in §3, a deformation which increases slowly with E_x can change discontinuously when E_x attains higher values; a similar situation can also be expected here. In the present case the existence or otherwise of discontinuity will be determined by additional parameters $B_\theta, \bar{B}_\theta, e_1, e_3$.

Remembering that equation (4.2) is valid only in the small deformation limit, it is still worth making an estimate of θ close to the bend transition. With $h = 0{\cdot}025$ cm, the anchoring is assumed to be equally strong at both surfaces; $B_\theta = \bar{B}_\theta = 0{\cdot}01$ cgs. Helfrich's formula [16, 25] can be used to derive an upper limit for e_3; using equation (3.1), $|e_3| < 4{\cdot}9 \times 10^{-4}$ esu. Let $E_x = E_{xc}$, $q \approx \pi/2$; then $\theta_s \sim -\sigma_e E_{xc}/\sigma_\theta \sim e_3 E_{xc}/2B_\theta$. From experiment, for $H = 3300$ emu, $E_{xc} \approx 1{\cdot}25$ esu (125 V between 0·33 cm; equation (3.6) yields a slightly higher value of 150 V). Then $\theta_s \approx 0{\cdot}03$ radian which is not large. It is therefore unlikely that flexoelectricity introduces much of a correction when the anchoring is strong and also when the stabilizing **H** is large enough, as in [12]. In case the anchoring is not strong enough or H is not high enough, flexoelectric intervention may not be negligible. This can be settled by more detailed experiments and rigorous calculations.

5. Conclusions: limitations of the present calculations

The continuum theory is used to study the effect of E acting parallel to the boundaries of a (positive χ_a) nematic sample in the presence of an additional **H**. Homogeneous deformation is studied at points well removed from domain walls under the rigid anchoring hypothesis ignoring free charges; effects of weak anchoring and flexoelectricity are briefly discussed. Results for bend geometry are in qualitative agreement with those of [11, 12] with the bend transition turning out to be discontinuous. The twist transition, however, is found to be continuous contrary to the experimental findings and theoretical predictions of [11]. In the present mathematical framework this seems to be caused by the orientation not distorting **E** in twist geometry. The calculations, which are set out *ab initio* to facilitate checking, seem to be correct and therefore point towards the need to bridge the gap between the present theory and experiment. It is suggested that while the twist transition is continuous in a sample having electrode gap much greater than thickness, the transition may turn discontinuous when the electrode gap is not large. The present mathematical model may be inadequate to understand this as all quantities may have to be taken to depend on more than one coordinate; such an effort lies outside the scope of this work. From a purely theoretical viewpoint one can envisage the possibility that when the lateral dimensions of the sample are large with respect to the thickness it may be possible to make the Freedericksz transition continuous by tilting the initial director orientation sufficiently away from the homeotropic in a plane normal to **E**.

Even the bend transition may become continuous under the additional action of **H** (of strength less than the magnetic bend threshold) applied along **E**. This may be of help in the determination of some material parameters or their ratios. Increasing K_1/K_3 or diminishing $\varepsilon_{\parallel}/\varepsilon_{\perp}$ can reduce (and even remove) the discontinuity at the bend transition. The former case may be relevant to discotic nematics for which divergence of that elastic ratio has been predicted [26] over certain temperature ranges. The second case may be of interest for nematics which exhibit strong dielectric relaxation; in such cases it may be possible to influence the discontinuity at the bend transition by changing the frequency of the applied electric field.

It is not possible to rule out the formation of domains containing homogeneous distortions of opposite parity above the bend threshold. A sufficiently strong **H** applied normal to the plates may be able to quench the deformation in all domains discontinuously; decreasing H may lead to the reformation of domains.

When **H** is applied obliquely at some angle ψ with $\mathbf{n}_0$, its effect can be different on different domains. In this case there is no electric threshold; one can only think of discontinuous changes that might occur between states of less and more deformation. It is found that such jumps occur whenever the major influence determining the nature of the distortion changes from **H** to **E** or vice versa. Decreasing E_x at constant H or increasing H at constant E_x may lead to a multidomain deformation changing irreversibly into a monodomain distortion; these changes may be continuous or discontinuous depending upon the relative strengths of **H** and **E** and also the tilt of **H**.

Discontinuities also occur when $\mathbf{n}_0$ is tilted away from the homeotropic towards **E**. In this case again an electric threshold does not exist and the deformation jumps from one state to another. These jumps can be interpreted to occur when the major control over the distortion changes from **E** to the sample planes (i.e. elastic torques) or vice versa. The discontinuity can be softened and even removed by tilting $\mathbf{n}_0$ sufficiently towards **E**. Calculations for these cases have been presented in the absence of **H**.

The rather general theoretical formulation allows one to conclude that it may not be realistic to treat the case of finite anchoring energy without including flexoelectricity. This is because flexoelectricity can cause a deformation to develop continuously in the bend geometry even when E_x is below the bend threshold. As this has an unmistakable bearing on experiments related to **E** induced biaxiality, the case of finite anchoring energy is treated briefly in the limit of low distortions. The conclusion of this preliminary investigation is that as long as anchoring is strong enough and the magnitude of the stabilizing **H** large enough the flexoelectric intervention may not be considerable; even when **E** is static. These conditions appear to be satisfied in [12] as the investigators have utilized an alternating electric field, so that flexoelectric effects are minimized. The flexoelectric distortion may be asymmetrical or even antisymmetric. More detailed investigations, theoretical and experimental, may be necessary to assess quantitatively the range of anchoring strength, H and E_x, over which flexoelectricity becomes important.

One of the limitations of the present effort lies in studying the problem from a purely static viewpoint. It must be remembered that changes in director orientation are generally accompanied by transient flow. It should be interesting to find out what effect such flow has on the occurrence of discontinuity. This assumes some importance because hydrodynamic flow can couple with orientation via viscous torques.

At several points in this work (see, for instance, figures 2 to 4) conclusions have been drawn regarding the nature of distortions in different domains. The continuum theory only predicts the possibility of domain formation without being able to deduce exactly the director orientation at the domain walls. It is also not straightforward to find out how many domains can actually form in the sample. A crude guess would be that the domain number must be even and related in some way to the aspect ratio of the sample (in the present case, $g/2h$). This is a quantity which does not figure in the calculations but which may be of importance in an experiment (in [11, 12], $g/2h \approx 7$).

It has been shown that under certain conditions the nature of deformation in one domain (say $\mathscr{D}_1$) may change over to another kind ($\mathscr{D}_2$). It has been tacitly assumed that when this happens the wall separating $\mathscr{D}_1$, $\mathscr{D}_2$ dissolves leaving the monodomain $\mathscr{D}_2$. This is only an assumption, not a prediction. In addition, only one degree of orientational freedom has been considered (splay–bend deformations). The change in the nature of distortion in a given domain is accompanied by a large jump in θ_m. Such a change, accompanied by a correspondingly high viscous torque, may result in the orientation changing along lines not predicted in this work; for instance, the director may buckle out of the splay–bend plane via a twist leading to the formation of a new set of domains. These points are certainly worth an experimental check.

An aspect not treated in this work is the occurrence of the modulated phase above the bend transition in the presence of a strong stabilizing **H**. The present work, dealing with homogeneous deformations, cannot shed much light on this subject except to indicate the possibility that the formation of the modulated phase may be facilitated by the increasing discontinuity at the bend transition caused by increasing the strength of the stabilizing **H**. Though simplified discussions are available in the literature [15, 16] there is really no substitute for a rigorous calculation to understand the formation of modulations; this is being attempted. Still, from previous experience, certain possibilities can be hinted at. It has been known experimentally [27] as well as theoretically [28, 29] that the stripe phase which sets in above the magnetic bend threshold close to the nematic–smectic A transition point can be suppressed by changing the tilt of **H** by a small angle. It would be interesting to consider two situations for the present case of the modulated phase [11, 12]. In the first, the modulated phase is produced above the discontinuous bend transition. Keeping E_x and H constant, **H** is rotated slowly away from its original direction (normal to the plates). The questions to be answered are: (i) How does the wave vector of the stripes change as **H** is rotated? (ii) Does the modulated phase disappear (continuously or otherwise) when **H** is rotated beyond a certain angle?

In the second experiment, **H** is applied to the uniformly aligned (homeotropic) sample at a small angle to the sample planes with $\mathbf{E} = 0$; now E_x is increased. The questions that can be raised are: (i) Does the homogeneous deformation change discontinuously from a low state to a high state? (ii) Does the modulated phase occur above this jump? (iii) Is there a critical angle of tilt of **H** beyond which the modulations do not occur at all?

Lastly it must be remembered that all calculations have been carried out by taking one domain at a time. This must be noted particularly in connection with the diagrams depicting hysteresis (figure 1, also [11]). Domains can be expected to form when deformation occurs above threshold. In an experiment when the voltage is subsequently diminished the decrease in distortion that is observed actually corresponds to a change in the total deformation in the sample including domains and walls. When the field is sufficiently reduced the distortion gets completely quenched and this enables an estimate of the width of the transition. A relevant question that arises is whether domain formation above threshold has any influence on the hysteresis width. Put in a different way the question reads: If the deformation above threshold can be made monodomain by some means and if the field is now reduced, will the same hysteresis width be observed as in the original experiment when multiple domain formation was allowed?

Let us note that there exists an experimental technique (see, for instance, [30]) for ensuring monodomain formation above a Freedericksz threshold. It is indicated later that an adaptation of such a method may be useful in providing an empirical answer to the above question.

In the bend geometry, starting with $\mathbf{n}_0$ along z and **E** along x, the deformation threshold E_u is noted. Now E_x is diminished and the point of extinction (E_d) of the distortion is determined. Then $\Delta E = E_u - E_d$ is the transition width with multiple domain formation.

In the second experiment, with $\mathbf{n}_0$ along z, **H** is applied in the xz plane at some angle (say 30°) with the z axis to produce a monodomain deformation. With **H** still acting on the sample, E_x is slowly enhanced to a value $> E_u$; the monodomain distortion is now sustained by the combined actions of **H** and **E**. Holding E_x constant, H is now slowly diminished to zero leaving the monodomain deformation controlled solely by E_x; in a way we have bypassed the threshold E_u by temporary use of a suitable **H**. If E_x is now diminished and the point of extinction E_d' determined, we can estimate $\Delta E' = E_u - E_d'$, the transition width when the deformation above threshold is monodomain. Whether ΔE and $\Delta E'$ are equal or not (within experimental error) will help answer the question whether the formation of multiple domains above threshold has any effect on the hysteresis width. Needless to say, a similar technique can be used in the twist geometry except that now $\mathbf{n}_0$ is along y and **H** is temporarily applied in the xy plane.

The author thanks a referee for useful comments which helped improved an earlier version of the manuscript.

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